

数学Ⅰ・数学Ⅱ・数学Ⅲ・数学Ⅳ・数学Ⅴ

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数学Ⅰ・数学Ⅱ 1の(1)・(2)に同じ。

2

$$(1) \quad \begin{cases} a_1 + a_2 = 2 \\ a_4 + a_5 = 54 \end{cases}$$

$a_n = ar^{n-1}$ (a : 初項, r : 公比) とすると

$$\begin{cases} a + ar = 2 & \cdots \cdots \textcircled{1} \\ ar^3 + ar^4 = 54 & \cdots \cdots \textcircled{2} \end{cases}$$

$$\textcircled{2} \text{より} \quad a(r^3 + r^4) = 54 \quad r^3(a + ar) = 54$$

$$\textcircled{1} \text{を代入して} \quad 2r^3 = 54 \quad r^3 = 27$$

$$r \text{ は実数より} \quad r = 3$$

$$\textcircled{1} \text{より} \quad a = \frac{1}{2}$$

$$\text{よって} \quad a_n = \frac{3^{n-1}}{2} \quad \cdots \cdots (\text{答})$$

(2) 初項から第 n 項までの和を S とする。

$$S = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{1}{2}(3^n - 1)}{3 - 1} = \frac{3^n - 1}{4}$$

$$\frac{3^n - 1}{4} > 500$$

$$3^n - 1 > 2000$$

$$3^n > 2001$$

$$3^7 = 2187, \quad 3^6 = 729$$

$$\text{よって} \quad n = 7 \quad \cdots \cdots (\text{答})$$

3

$$(1) \quad \overrightarrow{AB} = (-2, 2, 0), \quad \overrightarrow{AC} = (-1, 1, 2)$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = 4 \quad \dots\dots (\text{答})$$

$$\triangle ABC \text{ の面積} = \frac{1}{2} \sqrt{|\overrightarrow{AB}|^2 |\overrightarrow{AC}|^2 - (\overrightarrow{AB} \cdot \overrightarrow{AC})^2} = \frac{1}{2} \sqrt{8 \cdot 6 - 4^2}$$

$$= 2\sqrt{2} \quad \dots\dots (\text{答})$$

$$(2) \quad \overrightarrow{DH} = \overrightarrow{AH} - \overrightarrow{AD} = s\overrightarrow{AB} + t\overrightarrow{AC} - (1, 2, 4)$$

$$= (-2s - t - 1, 2s + t - 2, 2t - 4)$$

$$\overrightarrow{DH} \cdot \overrightarrow{AB} = 0 \text{ より}$$

$$(-2s - t - 1) \cdot (-2) + (2s + t - 2) \cdot 2 = 0$$

$$8s + 4t - 2 = 0$$

$$4s + 2t = 1 \quad \dots\dots \textcircled{1}$$

$$\overrightarrow{DH} \cdot \overrightarrow{AC} = 0 \text{ より}$$

$$(-2s - t - 1) \cdot (-1) + (2s + t - 2) \cdot 1 + (2t - 4) \cdot 2 = 0$$

$$4s + 6t = 9 \quad \dots\dots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ より} \quad s = -\frac{3}{4}, \quad t = 2 \quad \dots\dots (\text{答})$$

$$(3) \quad \overrightarrow{DH} = \left(-\frac{3}{2}, -\frac{3}{2}, 0 \right)$$

$$|\overrightarrow{DH}| = \sqrt{\left(-\frac{3}{2} \right)^2 + \left(-\frac{3}{2} \right)^2} = \frac{3\sqrt{2}}{2} \quad \dots\dots (\text{答})$$

四面体 ABCD の体積は

$$\frac{1}{3} \times 2\sqrt{2} \times \frac{3\sqrt{2}}{2} = 2 \quad \dots\dots (\text{答})$$